Definition of statistical results

December 16, 2019

1 Mean variables

χ_k, u and p are 3D instantaneous fields, respectively phase indicator function, vecolity and pressure. $\overline{\phi}$ is the time/plane average of ϕ. x is the streamwise direction, z is the wall-normal direction and y is the spanwise direction. Thus, the different lines in files "XXXX_Mean_variables.txt" are defined as:
vapour volume fraction $= \alpha_v = \overline{\chi_v}$
liquid volume fraction $= \alpha_l = \overline{\chi_l}$
liquid velocity $= u_{l,i} = \frac{\overline{\chi_l u_i}}{\overline{\chi_l}}$
vapour velocity$=u_{v,i} = \frac{\overline{\chi_v u_i}}{\overline{\chi_v}}$
liquid pressure$=$ $p_l = \frac{\overline{\chi_l p}}{\overline{\chi_l}}$
vapour pressure$=$ $p_v = \frac{\overline{\chi_v p}}{\overline{\chi_v}}$
Rij total$=$ $\overline{u'_i u'_j}$
Rij liquid$=\alpha_l \overline{u'_{l,i} u'_{l,j}}$
Rij vapour$=\alpha_v \overline{u'_{v,i} u'_{v,j}}$

2 QdM monofluid

the Averaged Navier-Stokes equation gives the content of files "XXXX_QdM_monofluid.txt" defined as:

$$0 = \underbrace{-\nabla \overline{P'}}_{\text{pressure gradient}} \underbrace{+(\overline{\rho} - \langle\rho\rangle)\,\mathbf{g}}_{\text{buoyancy}} \underbrace{-\beta \mathbf{e}_z}_{\text{source terms}} + \underbrace{\nabla \cdot \overline{\tau}}_{\text{viscous operator}} \underbrace{-\nabla \cdot \left(\overline{\rho \mathbf{u'} \otimes \mathbf{u'}}\right)}_{\text{Reynolds stresses}} + \underbrace{\overline{\sigma \kappa \mathbf{n}_v \delta^i}}_{\text{surface tension source term}} . \quad (1)$$

where each of the single-fluid variables is defined as a mixture of phase variables: $\phi = \sum_k \chi_k \phi_k$. Here χ_k is the phase indicator function, which is equal to 1 in phase k and 0 otherwise.β is a constant source term containing the spatially averaged weight of the mixture $\langle\rho\rangle\mathbf{g}$ and the driving pressure gradient ($\langle\phi\rangle$ is ϕ averaged over the whole domain). It corresponds to an imposed shear stress at the wall. σ is the surface tension, $\kappa = -\nabla \cdot \mathbf{n}_v$ is the local curvature, and $\mathbf{n}_v$ is the interface normal defined by $\nabla \chi_v = -\mathbf{n}_v \delta^i$, where δ^i is the Dirac impulse at the interface.

3 QdM vapour or liquid

Multiplying the Navier–Stokes equation in phase $k \in [l, v]$ by the indicator function χ_k and averaging it leads, at statistical equilibrium, to the integrated version of the phase momentum balance (this equation gives the content of files "XXXX_QdM_liquid.txt" and "XXXX_QdM_vapour.txt"):

$$-\underbrace{\overline{P_i'\nabla\chi_k - \tau_i\cdot\nabla\chi_k}}_{\text{Interfacial force}} = \underbrace{-\nabla\left(\alpha_k\overline{P_k'}^k\right)}_{\text{pressure gradient}} + \underbrace{\alpha_k\left(\rho_k - \langle\rho\rangle\right)\mathbf{g}}_{\text{buoyancy}}\underbrace{-\alpha_k\beta\mathbf{e}_z}_{\text{source term}} \tag{2}$$

$$+\underbrace{\nabla\cdot\alpha_k\overline{\tau_k}^k}_{\text{viscous operator}}\underbrace{-\nabla\cdot\left(\alpha_k\overline{\rho_k\mathbf{u_k'}\otimes\mathbf{u_k'}}^k\right)}_{\text{Reynolds stresses}}, \tag{3}$$

In this equation, $\overline{\phi_k}^k$ is the phase average defined as $\overline{\phi_k}^k = \overline{\chi_k\phi_k}/\overline{\chi_k}$. This formulation reveals the term corresponding to the averaged force exerted by the interface on phase k calculated as the residue of the RHS of the equation.

4 Rij transport equation

The Reynolds stress transport equation at the statistical equilibrium is:

$$\underbrace{\alpha_l R_{l,ib}\frac{\partial\overline{u_{l,j}}^l}{\partial x_b} + \alpha_l R_{l,jb}\frac{\partial\overline{u_{l,i}}^l}{\partial x_b}}_{\text{production}} = \underbrace{\alpha_l\overline{\frac{P_l'}{\rho_l}\left(\frac{\partial u_{l,i}'}{\partial x_j} + \frac{\partial u_{l,j}'}{\partial x_i}\right)}^l}_{\text{redistribution tensor}}\underbrace{-2\alpha_l\frac{\mu_l}{\rho_l}\overline{\frac{\partial u_{l,i}'}{\partial x_b}\frac{\partial u_{l,j}'}{\partial x_b}}^l}_{\text{dissipation}}\underbrace{-\frac{\partial}{\partial x_b}\left(\alpha_l\overline{u_{l,i}'u_{l,j}'u_{l,b}'}^l\right)}_{\text{molecular diffusion}} \tag{4}$$

$$+\underbrace{\frac{\partial}{\partial x_b}\left(\nu_l\frac{\partial\alpha_l R_{l,ij}}{\partial x_b}\right)}_{\text{turbulent diffusion}}\underbrace{-\frac{\partial}{\partial x_b}\left(\frac{\alpha_l}{\rho_l}\left(\overline{P_l'u_{l,i}'}^l\delta_{bj} + \overline{P_l'u_{l,b}'}^l\delta_{ib}\right)\right)}_{\text{pressure diffusion}}$$

$$\underbrace{-\frac{1}{\rho_l}\overline{\left(P_l'u_{l,j}'n_i + P_l'u_{l,i}'n_j\right)\delta^i}^l + \nu_l\left[\frac{\partial}{\partial x_b}\left(\overline{u_{l,i}'u_{l,j}'n_j\delta^i}^l\right) + \overline{\frac{\partial u_{l,i}'u_{l,j}'}{\partial x_b}n_b\delta^i}^l\right]}_{\text{interfacial production}}, \tag{5}$$

It describes the content of files "XXXX_Rij_transport_equation.txt". The production term corresponds to the energy injected at large scales owing to local shear. It is expected to exhibit behavior similar to that of single-phase flows even if the presence of bubbles can lead to stronger shear located in the vicinity of interfaces. The interfacial production corresponds to the energy injected at the bubble scale. The redistribution redistributes energy between the components through fluctuations in pressure. Lastly, they are dissipation and diffusion terms.